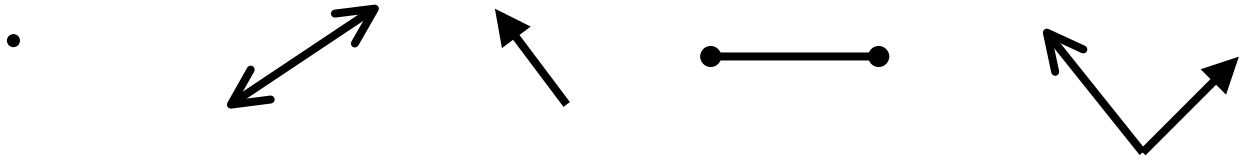
Geometry

Introduction: We live in a world of shapes and figures. Objects around us have length, width and height. They also occupy space. On the job, many times people make decision about what they know about geometric shapes. Carpenters measure angles to build a house. Engineers decide on angles in building a new highway. Contractors use angles and shapes to create beautiful designs as they lay ceramic tile.

Geometry is the study of shapes and sizes. The next few pages will review some basic geometry facts. Enjoy the short lesson on geometry.

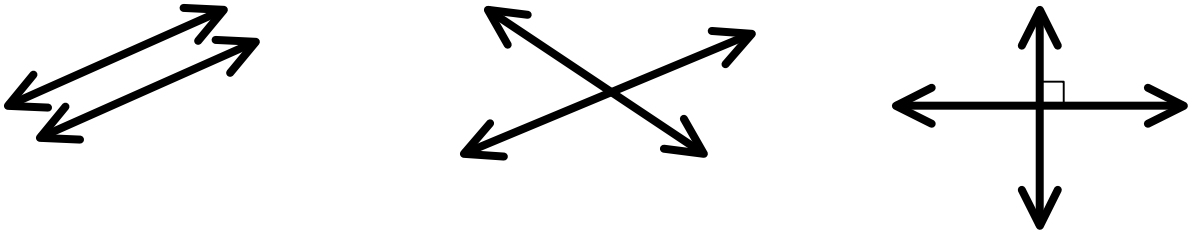
Part 1 Simple Definitions and Diagrams

All geometric figures are made up of at least one point and/or a combination of points.



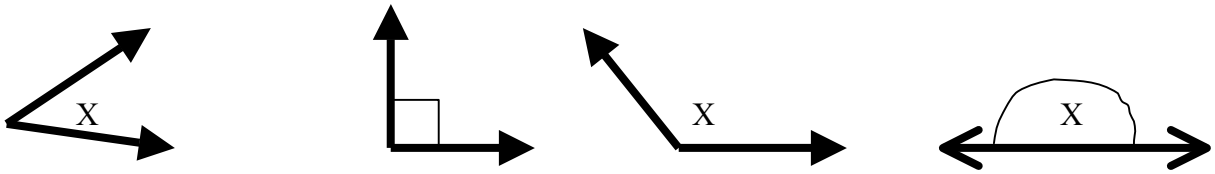
point line ray segment angle

Lines in a plane can be either parallel to each other or they can intersect in a point.



Parallel lines intersecting lines perpendicular lines (90 degree angle)

Angles are measured in degrees. Study the different kinds of angles below.



Acute angle
$0 < x < 90^\circ$

right angle
$x = 90$ degrees

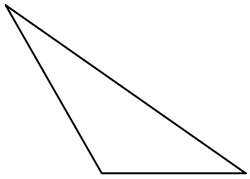
obtuse angle
$90^\circ < x < 180^\circ$

straight angle
$x = 180^\circ$

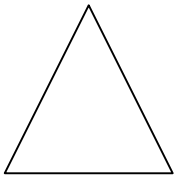
Complementary angles are two angles whose sum is 90°.
Supplementary angles are two angles whose sum is 180°.

Part 2 Triangles

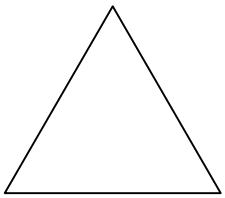
Triangles are polygons that contain three sides. Triangles can be classified according to the measure of their angles or the length of their sides.



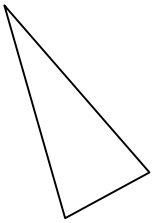
Scalene triangle
No sides equal



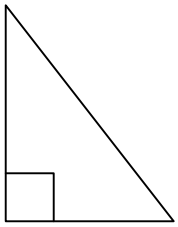
Isosceles triangle
2 sides equal



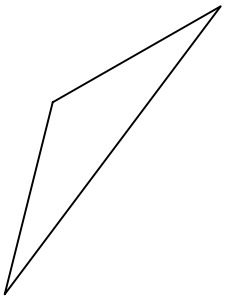
Equilateral triangle
3 sides equal



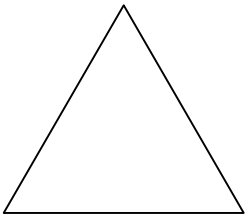
Acute triangle
all angles acute



Right triangle
one right angle

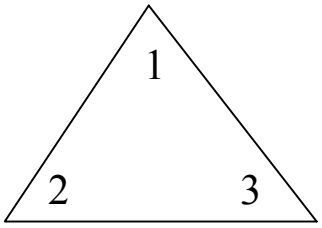


Obtuse triangle
one obtuse angle



Equiangular triangle
all angles are equal

The sum of the 3 angles in any triangle is 180°. Find the missing angles in each triangle.



∨ 1	∨2	∨3	
50°	70°	_?__	180 – (50+70) = 60
42°	64°	_?__	180 – (42+64) = 74

Part 3 Polygons

The word polygon comes from the Greek word meaning many angles. Polygons are closed figures made with straight-line segments. Polygons are named by the number of sides it contains. The following is a short list of the more common names of polygons according to the number of sides.

3 sides	triangle	8 sides	octagon
4 sides	quadrilateral	10 sides	decagon
5 sides	pentagon	12 sides	dodecagon

6 sides hexagon n-sides n-gon

Part 4 Perimeter Formulas

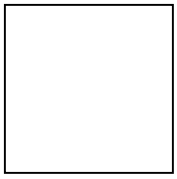
The distance around a region is called the perimeter. In the following formulas, l = length, w = width, s = side, and P = perimeter. The following gives the formula for the perimeter of a square and a rectangle.

Length

Width

Perimeter of a rectangle: $P = 2\boldsymbol{l} + 2\boldsymbol{w}$

Side



Perimeter of a square: $P = 4\boldsymbol{s}$

Find the perimeter of the following rectangles given the length and width.

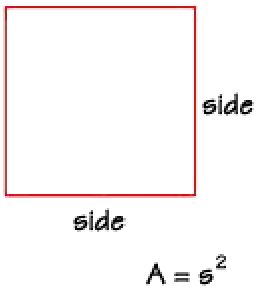
Length	Width	perimeter
2 ft	1 ft	2 x 2 + 2 x 1 = 4 + 2 = 6 ft
4 inches	3 inches	2 x 4 + 2 x 3 = 8 + 6 = 14 inches
12 cm	8 cm	2 x 12 + 2 x 8 = 24 + 16 = 40 cm
1.5 ft	0.5 ft	2 x 1.5 + 2 x 0.5 = 3 + 1 = 4 ft

Find the perimeter of the following squares given the length of a side.

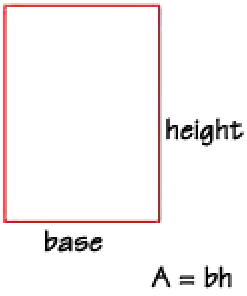
Length of a side	perimeter
5 ft	4 x 5 = 20 ft
6 inches	4 x 6 = 24 inches
14 cm	4 x 14 = 56 cm
1.5 ft	4 x 1.5 = 6 ft

Part 5 Area Formulas

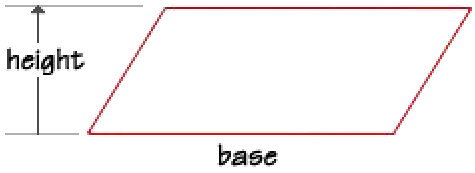
Area is always a positive number. It represents the number of square units needed to cover a figure. The following are formulas used to calculate area. Area is always calculated in squared units.



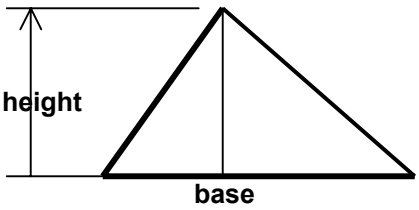
The area of a square is equal to the length of one side squared ($s^{2)}$ since both sides have the same length.
If the length of one side of a square is 4 cm, what is the area?
$A = s^2$ $A = 4\text{ cm} \times 4\text{ cm} = 16\text{ cm}^2$



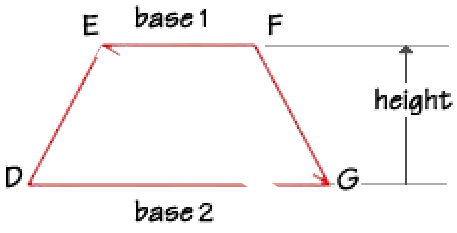
The area of a rectangle is equal to the product of the length of its base times its height. The base and height are often called the "length and width". Find the area of a rectangle if the height is 6 feet and the base is 2 feet.
$A = bh$ $A = 4\text{ ft} \times 6\text{ ft} = 24\text{ ft}^2$



The area of a parallelogram is equal to the product of the base times the height. The height is a line drawn from one vertex perpendicular to the line that contains the opposite side. Find the area of a parallelogram if the base is 23 cm and the height is 7 cm.
$A = bh$ $A = 23\text{ cm} \times 7\text{ cm} = 161\text{ cm}^2$

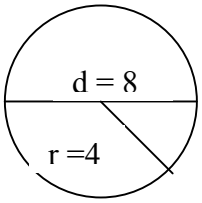


The area of a triangle is $\frac{1}{2}$ the base times the height. What is the area of a triangle with a base of 23 feet and a height of 16 feet?
$A = \frac{1}{2} bh$ $A = \frac{1}{2}(23)(16) = \frac{1}{2} \times 368 = 184\text{ ft}^2$

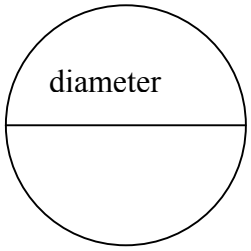


The area of a trapezoid is $\frac{1}{2}$ (base $_1$ + base $_2$) times the height. Find the area of a trapezoid that has a height of 8 inches, base $_1$ measures 7 inches and base $_2$ measures 12 inches.
$A = \frac{1}{2}(\text{base}_1 + \text{base}_2)\, h$
$A = \frac{1}{2}(7\text{ in} + 12\text{ in}) \times 8\text{ in} = \frac{1}{2}(19\text{ in}) \times 8\text{ in} =$
$\frac{1}{2} \times 8\text{ in} \times 19\text{ in} = 4\text{ in} \times 19\text{ in} = 76\text{ in}^2$

Part 6 **Circle Formulas**

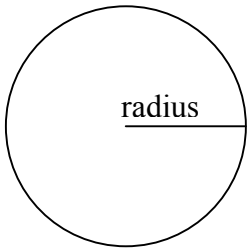


Remember: The radius of a circle is equal to ⌊ of the diameter. The radius of the circle to the right is 4.



The circumference of a circle is another name for perimeter of a circle. The formula to find the circumference of a circle is pi times the diameter. Find the circumference of a circle with the diameter of 4 inches.

$C = \neq d$ $C = \neq \langle 4 \text{ inches} = 3.14 \langle 8 \text{ inches} = 25.12 \text{ inches}$



The formula to find the area of a circle is pi times the radius squared. Find the area of a circle with a radius of 5 inches.

$A = \neq r^2$ $A = \neq \langle 5^2 = \neq \langle 25 = 3.14 \langle 25 = 88.5 \text{ in}^2$

Practice.

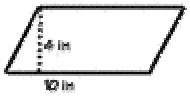
1. Find the area of each figure.

a.

square

$A=s^2$
$A=6^2$
$=36 \text{ cm}^2$

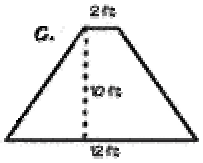
b.



parallelogram

$A=bh$
$A=10 \cdot 4$
$=40 \text{ in}^2$

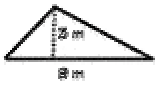
c.



trapezoid

$A = \lfloor (b_1 + b_2)h$
$A = \lfloor (2 + 12)10$
$A = \lfloor (14)10$
$A = 7 (10)$
$A = 70 \text{ ft}^2$

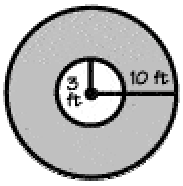
d.



triangle

$A=\frac{1}{2}bh$
$A=\frac{1}{2}(9 \cdot 3)$
$A=\frac{1}{2}(27)$
$=13.5 \text{ m}^2$

2. What is the area of the shaded region?



Answer: subtract the area of the smaller circle from the area of the larger circle.

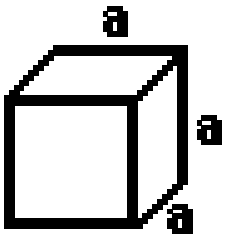
Large circle: $A=\pi r^2$ $A=3.14(10)^2$ $A=3.14(100)=314 \text{ ft}^2$

Small circle: $A=\pi r^2$ $A=3.14(3)^2$ $A=3.14(9)=28.26 \text{ ft}^2$

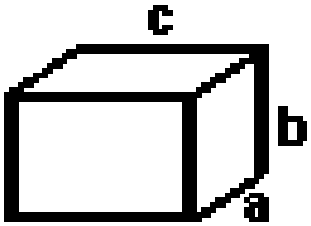
Now subtract: $314 - 28.26 = 285.74 \text{ ft}^2$

Part 7 Volume Formulas

Geometric solids are figures made up of plane polygons. In any prism, the volume is equal to the Base times the height. The Base could be a square, a circle, or any other polygon. Volume is calculated in cubic units.



The formula to find the volume of a cube is ***side*** times ***side*** times ***side***. Find the volume of a cube with each side 4 inches.
$V = a^3$ $V = 4^3 = 4 \times 4 \times 4 = 64$ in^3



The formula to find the volume of a rectangular prism is ***a*** times ***b*** times ***c.*** You can also refer to ***a, b and c*** as the width, height and length. Find the volume of a rectangular prism with ***a*** = 3 cm, ***b*** = 5 cm and ***c*** = 8 cm.
$V =$ ***abc*** $V = 3 \times 5 \times 8 = 120$ cm^3 or
$V =$ ***length*** x ***width*** x ***height*** $V = 3 \times 5 \times 8 = 120$ cm^3